

## Connections on Valuated Binary Tree and Their Applications in Factoring Odd Integers

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### Abstract

In this study, we look at the valuated binary trees' geometric relationships (parallelism, penetration, and connection) between their nodes. There are inherent links between nodes in various subtrees that may be discovered by calculating central lines and the distance between a node and a line. It is shown that a node may join a subtree using a parallel connection. Each node along the connection is a multiple of the root if the connection originates from a node that is a multiple of the subtree's root. As a result, factoring composite odd numbers on such connections is straightforward. The study presents a number of numerical tests that effectively factor large odd numbers with lengths ranging from 59 to 99 decimal digits using Maple software. To back up the new findings, the study gives a comprehensive mathematical explanation. The valuated binary tree is shown to provide an alternative to the integer factorization problem's lock once again. Relevant concepts include valued binary trees, number factorization, linkages, and parallel lines.

### Introduction

• Binary trees were first suggested as a tool for investigating odd integers greater than one in WANG's article [1]. The investigation that followed and the study itself discovered a number of novel characteristics. While [2] and [3] described the characteristics of

symmetric nodes and symmetric common divisors, articles [4] and [5] disclosed the genetic aspects of odd numbers. Furthermore, the periodic divisibility qualities along the left side-path or leftmost path of the tree were depicted in article [5]. People can now view integers from a new angle thanks to all these additional qualities, as discussed and investigated in paper [6]. • • Based on these novel characteristics, quick ways for factorising odd numbers have been discovered. To factorise an odd integer  $N = pq$ , for example, article [7] provided an  $O(\log_2 N)$  searching steps algorithm (or  $O((\log_2 N)^4)$  bit operations), where the divisor  $p$  satisfying  $1 \leq p \leq 2a - 1$  or  $2a + 1 \leq p \leq 2a - 1$ , article [8] showed a fast way for factorised big Fermat numbers, and article [9] provided a way to estimate the bounds of divisors for semiprimes or RSA numbers. It makes reasonable, then, to believe that the integer factorization conundrum may be solved by utilising the valuated binary tree. • • Of course, knowing the distribution of all the multiples of an odd integer  $p$  larger than 1 helps when factorising a composite odd integer with  $p$  as a divisor. The distribution of the multiples of the root  $p$  is important under description of the valuated binary tree,  $T_p$ , as discussed in works [4,5,6] and [7]. When a multiple, let's say  $m = \alpha p$  with odd number  $\alpha \geq 1$  and  $(\alpha, p) = 1$ , lies in the tree  $T_p$ , finding  $m$  is quite easy. • • be factorised by  $\{p \mid \gcd(m, p) = p\}$  as  $p$  is obtained by utilising  $\log_2 \alpha$  steps to trace upward from  $m$ . Since unfortunately • •

My Tp is running low. In most cases, it is evident that the study question would be how to connect to an inner descendent of Tp. In this work, such research is conducted. The paper first specifies many metric relations on the valued binary tree from the geometric point of view, and then uses these definitions to show that there is a certain class of odd integers that can be factorised in  $O(\log_2 N)$  searching steps. After that, it ascertains the converting relations of a tree from an outer node to an inner node. •• The paper is divided into five sections. This is the first section; the second is a citation of some earlier relevant preliminary work; the third contains some new definitions; the fourth contains new theorems and their proofs; the fifth and final section factorises the particular sort of odd integers.

## Preliminaries

The concepts, notations, and lemmas that have been defined, clarified, or proved in relevant previous publications and are essential for later explanations are referenced in this part. Here are also some novel findings along with their simple proofs

## Definitions and notations

A perfect complete binary tree, in which each node is given a value, is a binary tree with

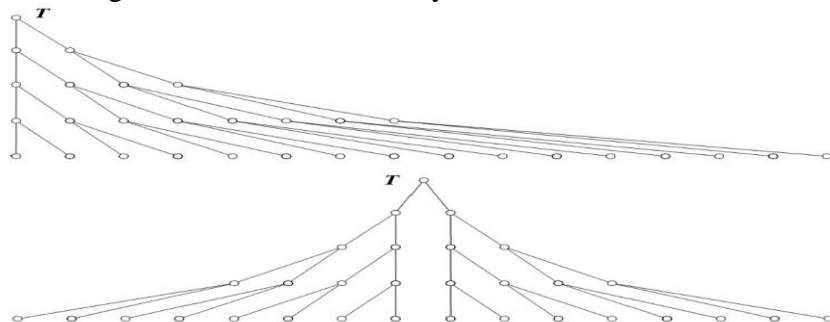


Fig. 1. Different layouts of a tree

values T. References to binary trees and its many components, including roots, nodes, fathers, left- and right-sons, and subtrees, may be found in data structure textbooks like Dinesh's handbook [10] (Dinesh P. Mehta, Sartaj Sahni, 2005). N must be an integer that is not divisible by 1. The left and right sons of the root are  $2N - 1$  and  $2N + 1$ , respectively, representing an N-rooted tree, which is a valued binary tree constructed recursively using N as its root. While there is a pathway that links each son to his father, there is no such pathway that links the two sons together. A direct route starts or ends at the root and connects a node to its immediate predecessor or successors. Direct ancestors include people like the father, grandfather, and so on. As the number of nodes on a route increases, its length also increases. All nodes that are at the same level are considered brothers. With  $N=3$ , we have the T3 tree.

## Geometric Relationships on a Tree

A valued binary tree is defined as a network of nodes and routes that begins with direct ancestors and continues through all generations of descendants. According to geometric principles, nodes should be arranged in rows and columns. One of the two possible layouts for the first five rows of a valued binary tree T is shown in Figure 1.

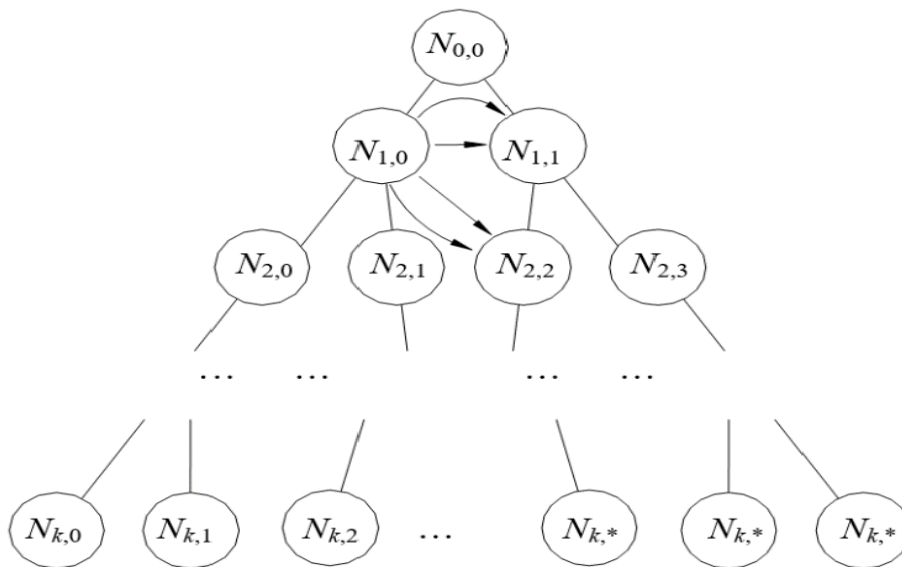
A column does not have a new name; a row is commonly referred to as a level. There is a gap between two nodes by definition. The tree is said to be equal-distanced when the distance between any two neighbouring levels and columns is equal. In scientific study, the equal-distanced tree is typically assumed; however, it is typically drawn to layout in an isosceles triangle, as seen in Fig. 2. The nodes are distributed parallel to one other across levels and columns, as can be observed. As will be discussed later, a valuated binary tree has additional geometric correlations in addition to parallelism.

**Fig. 2. Left-center line and right-center line**

Lemma 1 (P1) states that TN is a subtree of T3 for any arbitrarily large odd integer  $N \geq 3$ . If two odd numbers, X and Y, satisfy the conditions  $X > 3$ ,  $Y > 3$ , and  $X \neq Y$ , then TX and TY are obviously two different subtrees. As a result, a node  $x \in T3$  for a subtree TX with  $X > 3$  may be  $x \in TX$  or  $x \notin TX$ . It can enter TX when  $x \notin TX$ . Because of this, unless otherwise specified, the root of a subtree in later paper statements is always larger than 3. A stroll can, by definition, follow a path, a connection, or any combination of these. A walk's ordered array of all paths and connections makes up its trace, which has a length determined by the number of non-repeat nodes on it. Walking into subtree TN 1, 1, as shown in Fig. 3, for instance, allows node  $N_{1,0}$  to make at least four selective decisions:

• **Trace and penetration**

- Along trace  $N_{1,0} \rightarrow N_{0,0} \rightarrow N_{1,1}$  that is combined of path  $N_{1,0} \rightarrow N_{0,0}$  and path  $N_{0,0} \rightarrow N_{1,1}$ ;
- Along trace (connection)  $N_{1,0} \rightarrow N_{1,1}$ ;
- Along trace (connection)  $N_{1,0} \rightarrow N_{2,1}$ ;
- Along trace  $N_{1,0} \rightarrow N_{2,1} \rightarrow N_{2,2}$  that is combined of path  $N_{1,0} \rightarrow N_{2,1}$  and connection  $N_{2,1} \rightarrow N_{2,2}$ .



**Fig. 3. Traces of a walk**

If the trace of a walk is parallel to  $C_l$  or  $C_r$  of a tree, the walk is a parallel walk. A penetration is a walk whose trace has the shortest length. Obviously the penetration of a node into a tree is worth to investigate because it concerns something with the optimal problems of finding a shortest path.

- **Applications in Integer Factorization**

Property 7, Property 7\*, and Property 8\* indicate that an odd integer of the form  $(2^\alpha - 2^\beta + \gamma)p$  must be a

descendant of the p-rooted tree, where  $\gamma \geq 1$  and  $p > 3$  are positive odd integers,  $\alpha > \beta$ . This on the other hand

mean that an odd integer  $m$  that has a divisor of the form  $2^\alpha - 2^\beta + \gamma$  can be factorized very soon.

This section proves the related results.

- **Numerical experiments**

Statistical analyses are conducted using the Maple programme. You can find the experimental findings in Table 1. In the table, the 'Big Number N' column represents the large odd composite number that needs to be factorised. The 'nDigits' column indicates the number of decimal digits. The 'sDivisor' column indicates the found divisor of N. The 'Tsteps' column offers the theoretical calculation of the number of searching steps based on the previous corollaries, while the 'Rsteps' column records the actual searching steps recorded by the computer. It is clear that the theoretical stages are identical to the practical ones when it comes to searching. The Maple programmes are given in the appendix so that readers may have a better understanding of the algorithms. They may be tested by readers using the programmes.

**Table 1.**

Big Number N	nDigits	sDivisor	Tsteps	Rsteps
1361129467683753874933991060479210657 3720569303980406995753	59	80000000000000001239	126	126
1004336277661868922213726306090627668 58404681029709092356097	59	61897001964269013744 9562111	106	106
3697086064679224675734480111663181901 6393505518278570670458580224861	68	21729518917671154277 8874311843	126	126
1559155429592009364435823204937723814 2052981863686538114062107392351	68	91638919976965192288 826967713	126	126
2156795733372051183573360314936866748 15718346332418321765033807708157	69	12676506002282294014 96702681091	126	126
2760698538716225514973902344910793166 8458716142620601169954803000803329	71	16225927682921336339 1578010288127	126	126
1013453127459823122874618528109162771 39253536353869833593151553211936321	71	59565421307610088978 0265054949823	126	126
2607406049708142190423610481163987976 7654369539753705042663627621537194650 5523582892895109067	93	16225927682921336339 1578010288127	200	199
2734063405978764905465627783897026706 6753924081589598660106153660338579389 7980093024134417319198667	99	17014118346046923173 1687303715884105727	200	199
4679981866866785826334486242139186024 8509227422788545866476175829541061193 5326445796738078604349487	99	46597757852200185432 64560743076778192897	196	195

## 6 Conclusions and Future Work

Valued binary trees allow for the definition of connection and penetration, which link nodes outside of the tree to those within. This opens the door for the introduction of certain external properties into the tree, which in turn opens the door to the discovery of other node attributes. The study of trees is therefore broadened. The approaches are confirmed by the study in this publication. Both the literature and the results of this work show that for any class of exceptional odd integers, factorization is possible in a reasonable amount of time. Theoretical considerations and numerical experiments presented in this work further illustrate this point of view. Much remains fairly raw, and there is a lot that needs refining. To enhance the precision of the limits for the properties of  $\alpha$  and  $\beta$  in the three corollaries, more study is necessary, for example. Further investigation is necessary to elucidate these points. Because this study just paints a broad image of the linkages and penetrations, more research is required to completely comprehend its more complex elements. If only more young people would step up to the plate.

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